Complex foliations, dynamics and geometry

Universidade Federal Fluminense, Niterói, July 23-27, 2018

Monday, July 23rd

Title: About the analytic classification of two dimensional neighborhoods of elliptic curves

Speaker: Frédéric Touzet

Institution: Université de Rennes 1, France

Abstract: I will investigate the analytic classification of two dimensional neighborhoods of an elliptic curve with torsion normal bundle. Joint work with Frank Loray and Sergey Voronin.

Title: Characterization of generic projective space bundles and applications to foliations

Speaker: Carolina Araujo

Institution: IMPA

Abstract: The existence of sufficiently positive subsheaves of the tangent bundle of a complex projective manifold X imposes strong restrictions on X. In particular, several special varieties can be characterized by positivity properties of their tangent bundle. There are various notions of positivity for distributions on complex projective manifolds. In this talk we consider distributions having big slope with respect to curve classes, obtaining characterizations of generic projective space bundles in terms of movable curve classes. We then apply this result to investigate algebraicity of leaves of foliations, providing a lower bound for the algebraic rank of a foliation in terms of invariants measuring positivity. This is a joint work with Stéphane Druel.

Title: The geometric Bogomolov conjecture

Speaker: Serge Cantat

Institution: CNRS - Rennes University

Abstract: I will give a complete proof of the geometric Bogomolov conjecture. We shall see families of abelian varieties, foliations with holomorphic leaves, and groups of linear transformations acting on tori. (based on a joint work with Z. Gao, P. Habegger, and J. Xie)

Title: Minimal models of threefold foliations
Speaker: Calum Spicer
Institution: Imperial College, UK
Abstract: We will first give an introduction to some of the basic ideas of the minimal model program. We will then explain some recent progress on the minimal model program for foliations on threefolds along with applications to the study of foliation singularities and the study of foliations with trivial canonical class. Joint work with Paolo Cascini.

Tuesday, July 24th

Title: The role of Puiseux characteristics in the local Poincaré problem
Speaker: Javier Ribón
Institution: Universidade Federal Fluminense, Brasil
Abstract: We provide a lower bound for the multiplicity of a germ of complex foliation in dimension two in terms of Puiseux characteristics of an irreducible invariant curve. This is a joint work with José Cano and Pedro Fortuny.

Title: Structural stability for algebraic foliations on the complex projective plane
Speaker: Bertrand Deroin
Institution: CNRS – Université Cergy-Pontoise
Abstract: I will prove the existence of a component of stability in the moduli space of algebraic foliations of degree 2 of the complex projective plane. I will also report on the history of this problem. This is based on a joint work with Aurélien Alvarez.

Title: A remark on the Fatou sets of foliations of $\mathbb{C}P^2$
Speaker: Taro Asuke
Institution: Graduate School of Mathematical Sciences, University of Tokyo
Abstract: Fatou sets of foliations of $\mathbb{C}P^2$ are firstly introduced by Ghys, Gomez-Mont and Saludes in the case where every singularity is of Poincaré type [4]. Another definition, which is involved but naive, is introduced by the speaker, and Fatou sets are defined for foliations of $\mathbb{C}P^2$ with isolated singularities which are not necessarily of Poincaré type [1], [2]. One of the significant properties of the Fatou sets is that they admit invariant transverse metrics of certain regularity. A partial converse has been known if singularities are of Poincaré type [1]. In this talk, I will explain the same holds in the general case. This will help to find Fatou components, particularly when foliations are given by logarithmic one-forms.

References

[1] T. Asuke, *A Fatou-Julia decomposition of transversally holomorphic foliations*, Ann. Inst. Fourier (Grenoble) **60** (2010), 1057–1104.

[2] T. Asuke, *On Fatou-Julia decompositions*, Ann. Fac. Sci. Toulouse **22** (2013), 155–195.

[3] T. Asuke, *On Fatou–Julia sets for foliations of surfaces*, preprint in preparation.

[4] E. Ghys, X. Gomez-Mont, J. Saludes, *Fatou and Julia components of transversely holomorphic foliations*, Essays on geometry and related topics, vol. 1,Monogr. Enseign. Math. **38** (2001), Enseignement Math., Geneva, 287–319.

Title: Local dynamics of skew-parabolic product maps.
Speaker: Liz Vivas
Institution: Ohio University, EUA
Abstract: We consider the map $F(x,y) = (f(x), g(x,y))$ with a fixed point at the origin and tangent to the identity. We identify different regions where the dynamics vary differently and give explicit conjugacy maps to the translation in different 'stable/unstable' domains and 'stable/unstable' parabolic curves.

Title: Foliations and webs with continuous symmetries on complex projective surfaces
Speaker: David Marín
Institution: Universitat Autònoma de Barcelona
Abstract: We will describe the structure of foliations and webs on complex projective surfaces which are invariant by a germ of birational flow. We will discuss in detail the case of the projective plane, characterizing planar projective webs with many infinitesimal symmetries. This is a joint work with Marcel Nicolau.

Wednesday, July 25th

Title: Isotriviality results for families given by nonsingular foliations
Speaker: Ekaterina Amerik
Institution: Université Paris-Sud, France
Abstract: Viehweg and Zuo obtained several results concerning the moduli number in smooth families of polarized varieties with semi-ample canonical class over a quasiprojective base. These results led Viehweg to conjecture that the base of a family of maximal variation is of log-general type, and the conjecture has been recently proved by Campana and Paun. From the "opposite" side, Taji proved that a smooth projective family over a special (in the sense of Campana) quasiprojective base is isotrivial. We extend Taji's theorem to quasismooth families, that is, families of leaves of compact foliations without singularities. This is a joint work with F. Campana.

Title: Positive neighborhoods of curves
Speaker: Paulo Sad
Institution: IMPA, Brasil
Abstract: We discuss the possibility of embedding 2-dimensional neighborhoods of curves into the projective plane.

Title: Teichmüller and Kuranishi
Speaker: Laurent Meersseman
Institution: Université d' Angers, France
Abstract: Let M be a compact oriented smooth manifold admitting complex structures. In this talk I will explain how stack theory can be used to handle its Teichmller space, that is the space of complex structures on M up to biholomorphisms smoothly isotopic to the identity. I will define this Teichmller stack but also a Kuranishi stack coming from classical Kodaira-Spencer deformation theory which encodes the local structure of the Teichmller stack up to a discrete part. I shall finally explain that this discrete part is different in the Khler and non-Khler setting.

Thursday, July 26th

Title: Differential Galois theory for connections with parameters and isomonodromic deformations
Speaker: David Blázquez-Sanz
Institution: Universidad Nacional de Colombia - Sede Medellín
Abstract: In this talk we give a geometric presentation of the Picard-Vessiot theory with parameters proposed by Landesman, Singer and Cassidy. In this geometric framework Galois groups are differential algebraic subgroups of the group of gauge symmetries of the connection. We see that, for connections with simple structural group, the Galois group with parameters is determined by the isomonodromic deformations and vice-versa. This is an application of Cassidy's classification of Kolchin closed differential subgroups and Morimoto-Kiso's theorem on intransitive Lie algebra sheaves. We apply it to the Schwartzian equation and compute the differential Galois group with parameters of Gauss' hypergeometric equation. This is an ongoing joint project with G. Casale (U. Rennes 1), and J. S. Díaz Arboleda (UNAL - Medellín).

Title: Deformations of exterior differential ideals and applications.
Speaker: Fernando Cukierman.
Institution: University of Buenos Aires.

Abstract: We plan to expose the basic elements of the deformation theory of exterior differential ideals on a smooth algebraic variety over the complex numbers. The applications refer to stability results for some types of ideals. Work with César Massri.

Title: Local uniformization of codimension one foliations
Speaker: Miguel Fernández-Duque
Institution: Universidad Autónoma de México

Abstract: In 1940 O. Zariski obtained Local Uniformization on algebraic varieties. Using this result, which is true in arbitrary dimension, in 1944 he obtained Resolution of Singularities in dimension three. It was twenty years later, in 1964, when H. Hironaka published his celebrated paper obtaining Resolution of Singularities in any dimension.

In the case of codimension one foliations, we only know reduction of singularities results in lower dimensions. In fact, only in dimension two (A. Seidenberg 1968) and three (F. Cano 2004). Following the ideas of Zariski, in this talk we present the result of Local Uniformization for codimension one foliations defined in ambient spaces of arbitrary dimension. This is a joint work with F. Cano.

Title: Separating holonomy of uninterrupted nodal components
Speaker: Marianna Ravara Vago
Institution: Universidade Federal de Santa Catarina, Brasil

Abstract: It is known that a nodal singularity in dimension two locally divides the set of leaves of a foliation. We wish to see if the same property holds for uninterrupted nodal components, which are the generalization of nodal singularities for a codimension one foliation in dimension three.

Title: Degenerate singular points of $\mathbb{C}^2$-actions on complex surfaces
Speaker: Helena Reis
Institution: Universidade do Porto, Portugal

Abstract: We classify degenerate singular points of rank 2, $\mathbb{C}^2$-actions on complex surfaces. Here the $\mathbb{C}^2$-action is said to be of rank 2 if there is an open orbit. Whereas this problem is related to Ghys-Rebelo classification of germs of holomorphic semicomplete vector fields in dimension 2, the case of rank 2, $\mathbb{C}^2$-actions is far more rigid and closely related to integrable systems. The corresponding classification also raises a number of interesting questions concerning the realization of the corresponding models on suitable surfaces (not necessarily compact). The realization problem will also be discussed in the talk and, if time permits, we will provide some new results and examples along these directions.

Friday, July 27th

Title: Characteristic directions of tangent to the identity biholomorphisms in dimension two.
Speaker: Lorena López
Institution: Universidad de Alcalá de Henares, Spain
Abstract: Let F be a germ of a two-dimensional tangent to the identity biholomorphism of order $k+1$. We will show that every characteristic direction of F supports either a curve of fixed points or k parabolic stable manifolds. This is a joint work with Rudy Rosas.

Title: On the dynamics of discrete subgroups of PSL(3,C)
Speaker: Pepe Seade
Institution: UNAM, México
Abstract: The transverse pseudogroup plays an important role when studying holomorphic foliations. A lot has been done in this sense concerning cxdimension 1 foliations. In particular it is interesting to study foliations whose transverse pseudogroup actually is a pseudo group of the projective group PSL(2,C). In this talk we look at discrete subgroups of PSL(3,C).

Title: The groupoid generated by PSL(2,C) and the exponential
Speaker: Daniel Panazzolo
Institution: Université de Haute Alsace, France
Abstract: I will expose a recent result on the structure of the groupoid of germs of diffeomorphism of the Riemann sphere generated by the Moebius transformations and the exponential. As a byproduct, I show that the subgroup of Homeo($\mathbb{R}, +\infty$)(i.e. the germs of real homeomorphism fixing the infinity) which is generated by the positive affine maps $x \mapsto ax + b$ (with $a > 0$), the exponential $x \mapsto \exp(x)$ and the logarithm $x \mapsto \ln(x)$ is isomorphic to a HNN extension (with the exponential playing the role of the stable letter).